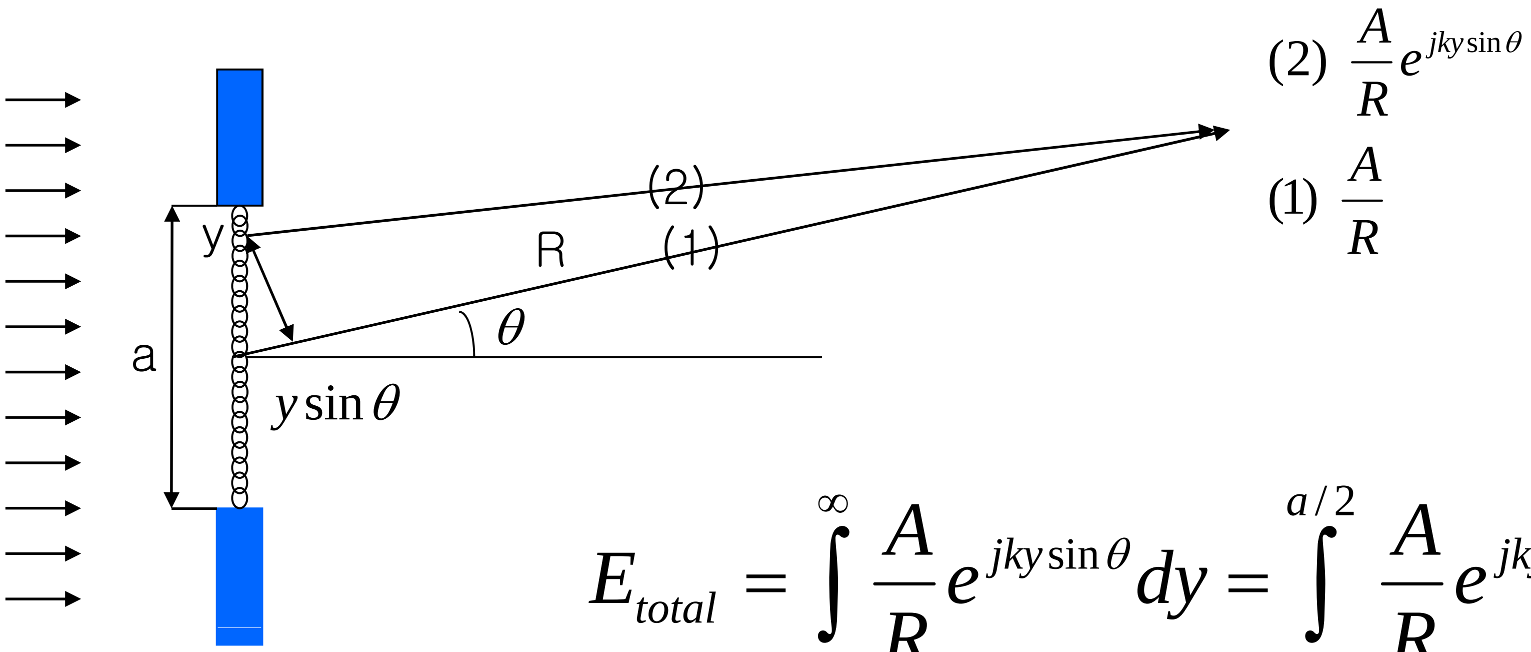
Lect. 9: Diffraction



$$E_{total} = \int_{-\infty}^{\infty} \frac{A}{R} e^{jky\sin\theta} dy = \int_{-a/2}^{a/2} \frac{A}{R} e^{jky\sin\theta} dy$$

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$$E_{total} = \int_{-a/2}^{a/2} \frac{A}{R} e^{jky\sin\theta} dy$$

Let $y' = jky\sin\theta$ => $dy' = jk\sin\theta dy$

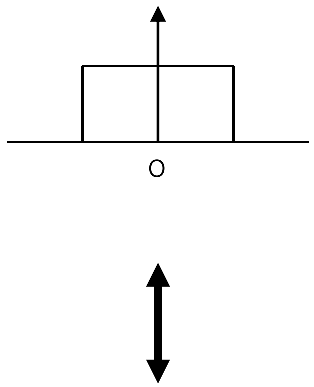
$$E_{total} = \int_{-a/2}^{a/2} \frac{A}{R} e^{y'} \frac{dy'}{jk\sin\theta} = \frac{A}{R}\frac{1}{jk\sin\theta}\left(e^{jk\frac{a}{2}\sin\theta} - e^{-jk\frac{a}{2}\sin\theta}\right)$$

$$= \frac{A}{R}\frac{2j}{jk\sin\theta}\sin(k\frac{a}{2}\sin\theta)$$

$$= \frac{2A}{R}\frac{\sin(k\frac{a}{2}\sin\theta)}{k\sin\theta}$$

$$E_{total}(0) = \frac{2A}{R}\frac{\cos(k\frac{a}{2}\sin\theta)k\frac{a}{2}\cos\theta}{k\cos\theta} = \frac{2A}{R}\frac{a}{2}$$

$$\therefore \frac{E_{total}(\theta)}{E_{total}(0)} = \frac{\sin(k\frac{a}{2}\sin\theta)}{k\frac{a}{2}\sin\theta} = \frac{\sin\psi}{\psi} \qquad \left(\psi = k\frac{a}{2}\sin\theta\right)$$



sinc function

Fraunhoffer Diffraction: R>>λ

FT relationship

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Remember $$f(t) = \int_{-\infty}^{\infty} F(f) e^{j2\pi ft} df$$

$$E_{total}\left(\frac{\sin\theta}{\lambda}\right) = \int_{-\infty}^{\infty} \frac{A(y)}{R} e^{j2\pi y \frac{\sin\theta}{\lambda}} dy$$

$$\mathrm{f}(t) \quad \Leftrightarrow F(f)$$

$$E\left(\frac{\sin\theta}{\lambda}\right) \Leftrightarrow A(y)$$

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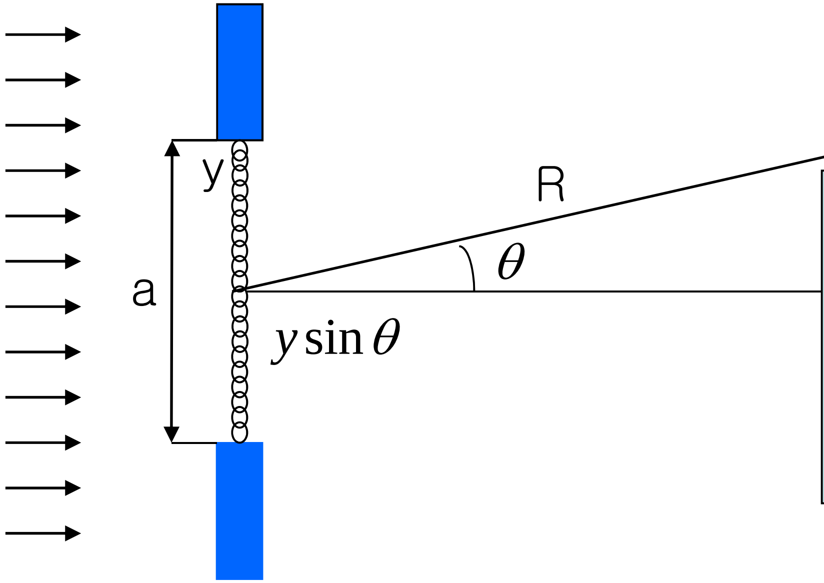
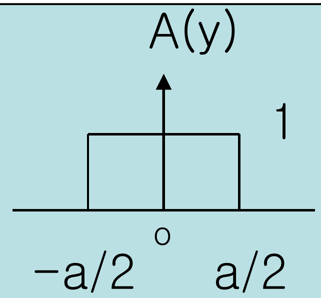


$$\frac{E(R,\theta)}{E(R,0)} = ?$$



FT

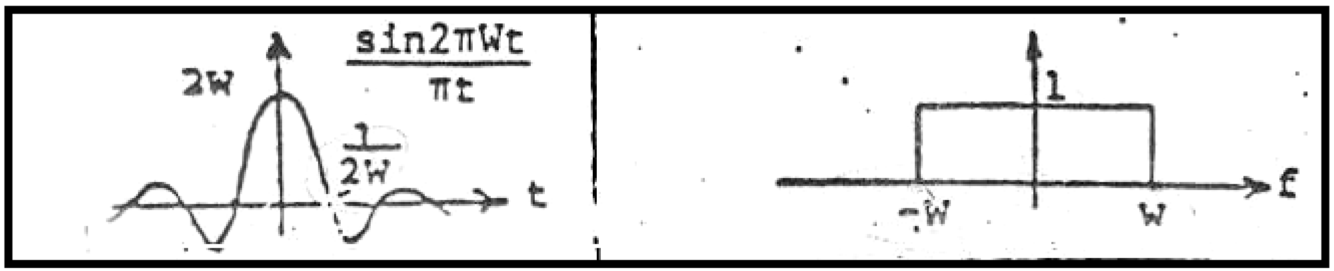
$$\frac{1}{R}\frac{\sin(2\pi\frac{a}{2}\frac{\sin\theta}{\lambda})}{\pi\frac{\sin\theta}{\lambda}}$$

$$\frac{1}{R}\frac{\sin(2\pi\frac{a}{2}\frac{\sin\theta}{\lambda})}{\pi\frac{\sin\theta}{\lambda}} = \frac{1}{R}\frac{\sin(\frac{ka}{2}\sin\theta)}{\frac{k}{2}\sin\theta}$$

Since $E(\theta=0)=\frac{a}{R}$,

$$\frac{E(\theta)}{E(0)} = \frac{\sin(\frac{ka}{2}\sin\theta)}{\frac{ka}{2}\sin\theta}$$

f(t) (From Signals textbook) F(f)



W.-Y. Choi

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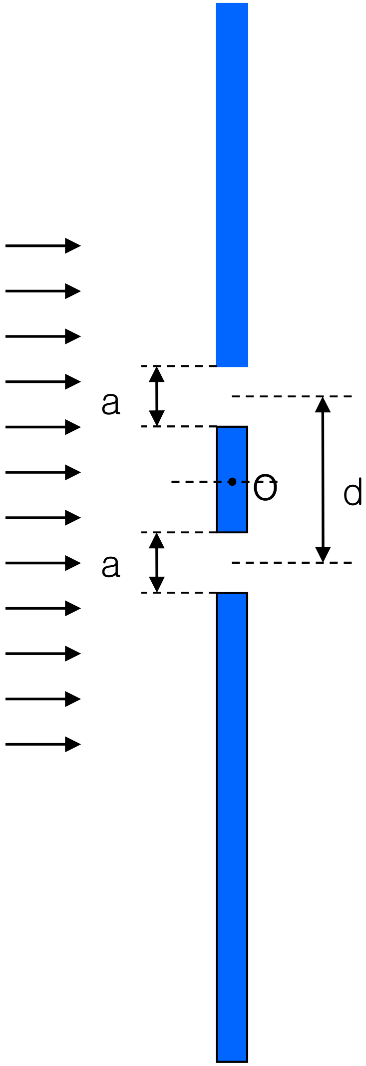

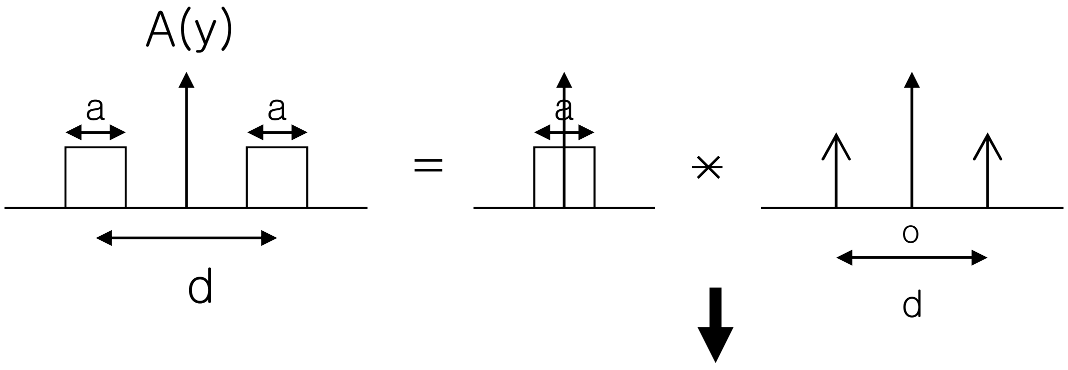

$$\frac{1}{R}\frac{\sin(2\pi\frac{a}{2}\frac{\sin\theta}{\lambda})}{\pi\frac{\sin\theta}{\lambda}} \times e^{j2\pi\frac{d}{2}\frac{\sin\theta}{\lambda}} + e^{-j2\pi\frac{d}{2}\frac{\sin\theta}{\lambda}} = 2\cos(2\pi\frac{d}{2}\frac{\sin\theta}{\lambda})$$

$$\frac{E(\theta)}{E(0)} = \cos(k\frac{d}{2}\frac{\sin\theta}{2})\frac{\sin(\frac{ka}{2}\sin\theta)}{\frac{ka}{2}\sin\theta}$$

Homework: Prob. 3 in 2002 Test 1 (Optional)

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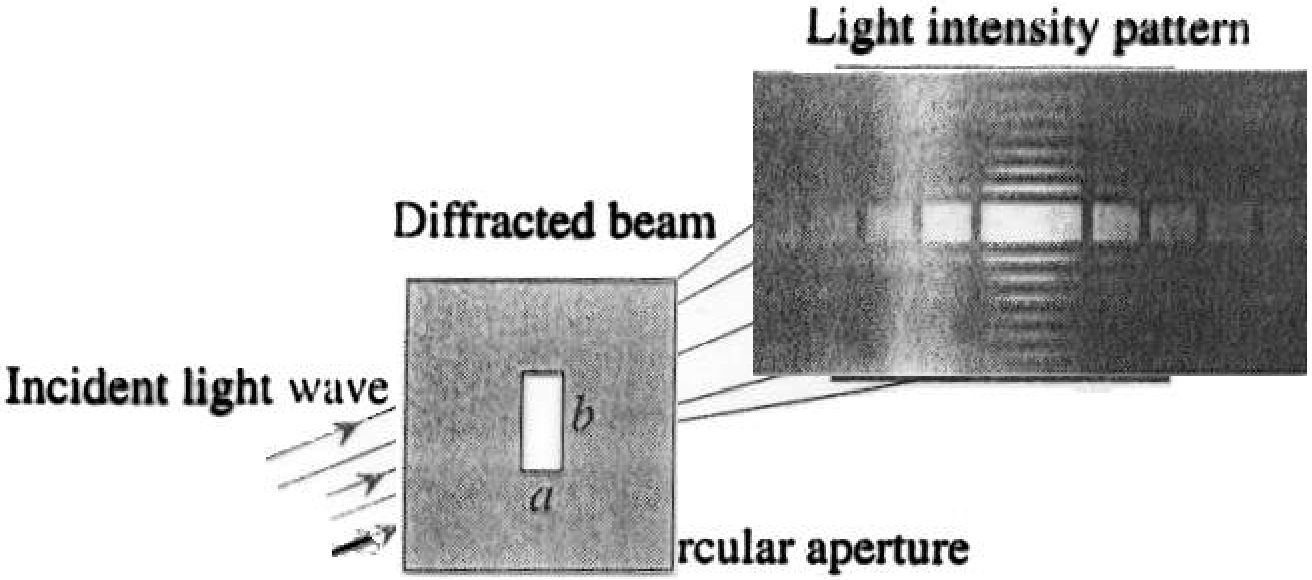

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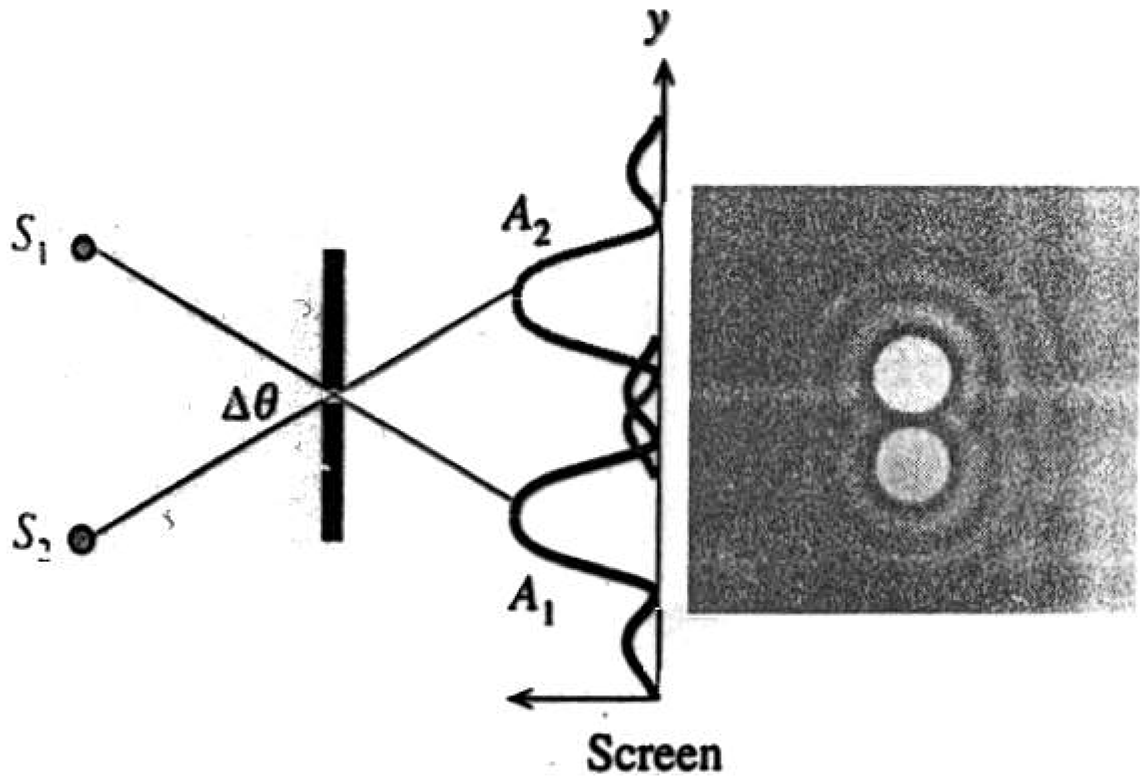

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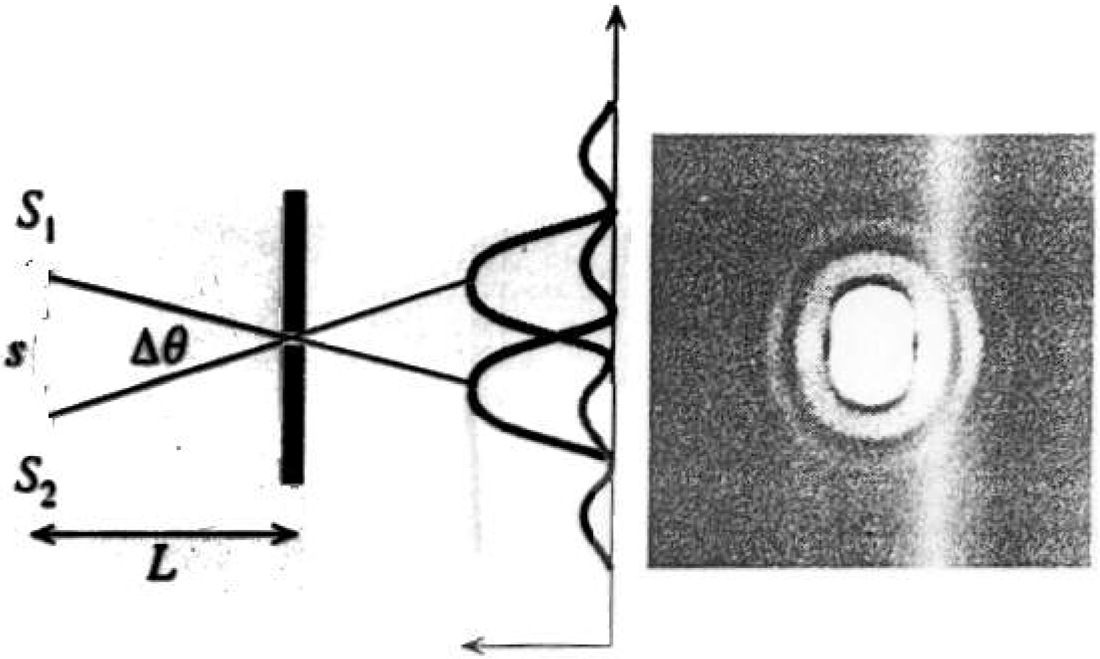

$$\Delta\theta \sim \frac{1}{a}$$

Lect. 9: Diffraction

In our analysis, diffraction is observed at locations having same R.

In reality, observation is made at a flat surface. Consequently, there is additional phase shifts.

➔ More complicated analysis required

Lect. 9: Diffraction

Homework: Optional

Two point light sources are located as shown below. We are interested in far-field pattern produced by the interference of these point sources. each of cases given below, sketch the magnitude of E(Θ)/E(0). For sketch, use sin(Θ)/λ as the x-axis. On the sketch, clearly indicate locations of the max. and min. magnitudes.

(a) Two source are located near origin and E-fields from two sources are phase when they are produced at the source.

(b) Two source are located near origin and E-fields from two sources out-of-phase when they are produced at the source.

(c) Same as in (a) but the location of sources are shifted by d.